Here There Be Dragons: Considering the Right Tail in Risk Management

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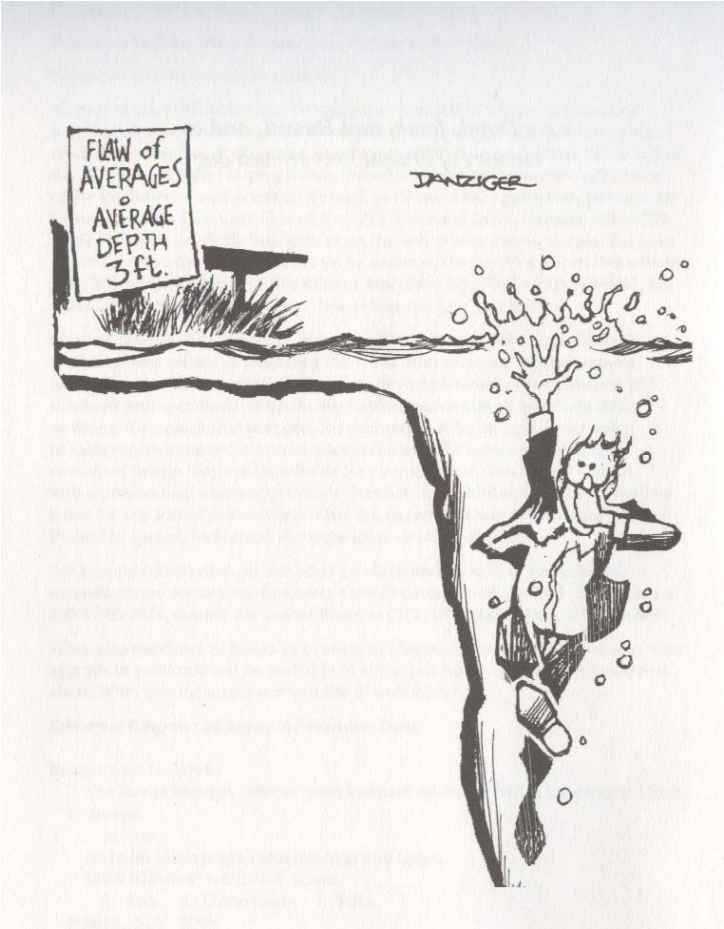
Introduction

- **"Do not attempt to cross a river based on the knowledge that it is on average four feet deep"**
  - **Nassim Taleb, author of *Fooled by Randomness* and *The Black Swan***

- **Risk management is important because we need to guard against the potential for adverse events**



Risk Measurement

- **"Western Europe conquered the world because of a technological revolution that started because of attempts to measure the world." – Phillipe Jorion**
  - **Similarly, effective risk measurement will lead to better project management**
- **Risk management for DoD and NASA agencies focuses primarily on funding projects to a single percentile.**
  - **NASA policy explicitly mentions 70% and 50%**
  - **This is known as "Value at Risk" and is prevalent in the banking and financial industry**

$$\boldsymbol{VaR_\alpha = inf\{l \in \mathbb{R}: P(L > l) \leq 1 - \alpha\}}$$

Value at Risk (VaR)

- **VaR is a percentile of a cost risk distribution**
  - **VaR funding at the 70th percentile means that there is a 30% chance of final project cost exceeding the funded amount (budget)**
- **VaR has some merits**
  - **Common measure, can be used to compare different projects and programs**
  - **Easily understood by senior managers and decision makers**

VaR and Risk Management

- However risk management does not stop at a specific percentile
  - What happens when the 70th percentile is exceeded?
- Any effective risk management policy should prescribe what happens after a bad event occurs
  - With VaR we only know when a bad event has occurred
  - Funding at a relatively low level means we have no true sense of the real risks that should be guarded against

"Here There Be Dragons"

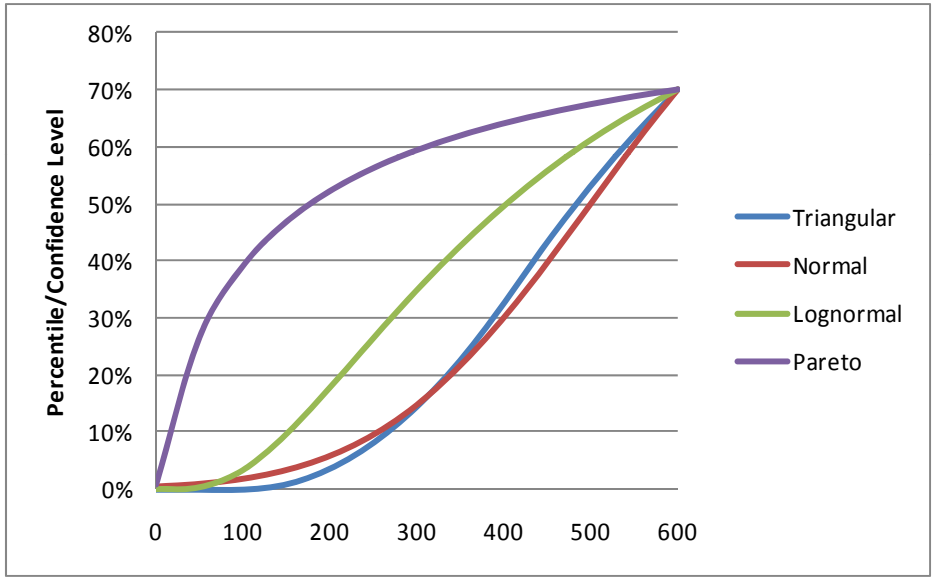
- **On old maps, uncharted territory was often marked with monsters or dragons**

A section of the Carta Marina by Olaus Magnus, 16th Century.

- **In the same way, VaR (percentile) funding gives us no sense of the risks in the tail of the cost risk distribution**

Distribution Comparison

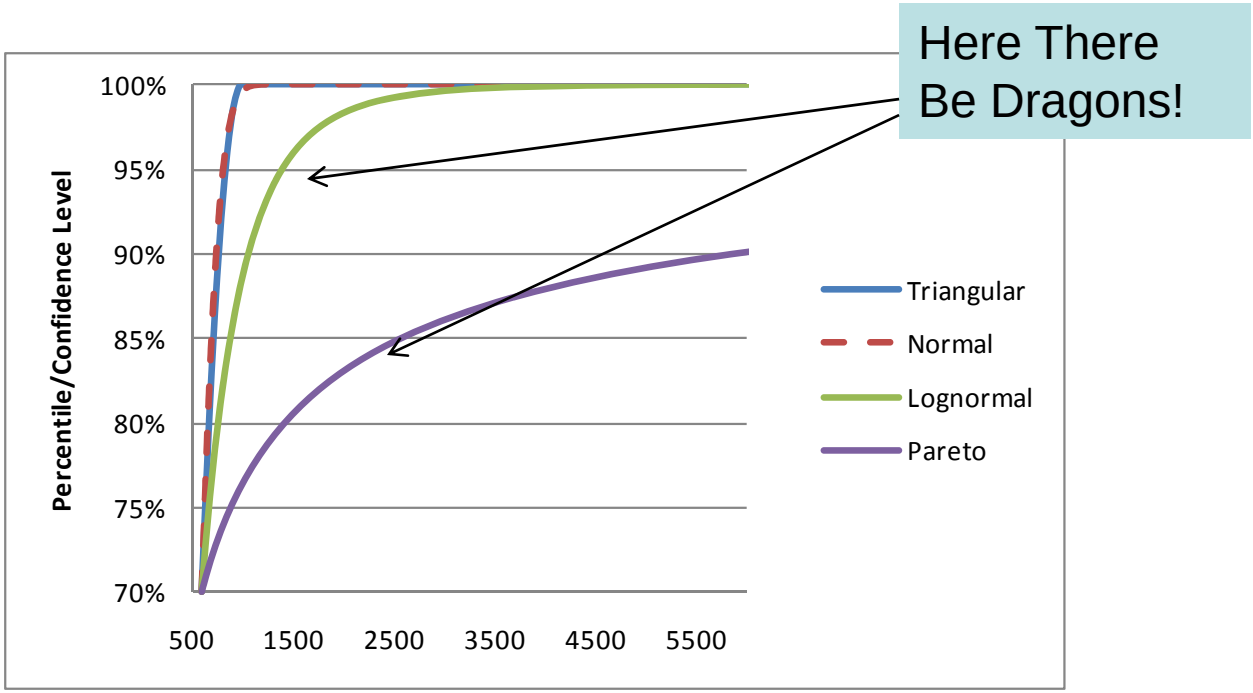
- **Example of four different distributions with the same 70th percentile**



- **Should we expect the tails to be similar?**

Distribution Comparison – Tail Behavior

- **The normal and triangular distributions have similar tails, but the lognormal and Pareto have much fatter right tails**



Distribution Comparison – Tail Behavior (Table)

Percentile	*Triangular*	*Normal*	*Lognormal*	*Pareto*
70th	$600	$600	$600	$600
80th	$680	$660	$770	$1,430
90th	$770	$750	$1,070	$5,880
95th	$840	$820	$1,410	$23,750
99th	$930	$950	$2,350	$600,000

- **There is significant tail risk for the lognormal and the Pareto**
- **The 99th percentile for the Pareto is 1,000 times greater than the 70th percentile**
  - **Risk seen in financial markets and in catastrophic risks like natural disasters**
- **Four different projects that follow the four different risk profiles seen in this example should clearly have different risk management approaches but 70th percentile funding indicates the same risk management approach for all four!**

Percentile Budgeting is Not Risk Management!

- **Percentile budgeting ≠ effective risk management policy**
- **Suppose that you are shopping for a new car. You mention that safety is your top concern. The salesman says he has a great, safe car available. You ask about the air bags. Do they work? The salesman answers, "Of course! Seventy percent of the time they work fine. Only 30% of the time, the air bags will fail to deploy." Would you buy such a car? Hedge fund manager David Einhorn has stated "Risk management is the air bag that must always work, but only in the multi-sigma event where you have an accident." (Ref. 7)**

Risk management is the air bag that must always work, but only in the multi-sigma event where you have an accident.

Percentile Budgeting is *Not* Risk Management!

- **As Nassim Taleb, author of The Black Swan, has stated, "You're worse off relying on misleading information than on not having any information at all. If you give a pilot an altimeter that is sometimes defective he will crash the plane. Give him nothing at all and he will look out the window." (Ref. 9)**
- **Average boring laid-back events are not what we should be safe-guarding against, but that's what 70th percentile budgeting provides**

Percentile budgeting is misleading information.

Thinking Coherently About Risk

- A risk measure is a single number that is used to represent cost risk for a project or program
  - The variance of the distribution is a risk measure
    - Quantifies the spread in the cost distribution
  - Value at risk is also risk measure and there are many others.
- What properties should a risk measure have?
- Artzner et al. have proposed the notion of coherent risk measures (Ref. 11)
  - Four properties
    - Subadditivity
    - Monoticity
    - Positive homogeneity
    - Translation invariance.

Coherent Property: Subadditivity

- **One property important for a risk measure is that when two random variables are combined the risk measure of the portfolio should be no riskier than the sum of the individual random variables' risk measures. That is, for any risk measure ρ, we should have that**

$$\rho(X+Y) \leq \rho(X)+\rho(Y)$$

Coherent Property: Monotonicity

- **Another desirable property for risk measures is that if one cost (X) is always less than a second cost (Y) when considered together, then the risk measure of X should be less than the risk measure for Y**
- **For example if the cost of thermal control is lower in every circumstance than the structures cost, then the 70th percentile of the cost risk distribution should be higher for the structures than for thermal control**
- **This is the property of monotonicity:**

$$X \leq Y \text{ for all possible outcomes } \Rightarrow \rho(X) \leq \rho(Y)$$

Coherent Property:
Positive Homogeniety

- **A risk measure should be invariant of the currency in which the risk is measured, or whether cost is accounted for in thousands or millions of dollars**
- **Also it means that an increase or decrease in exposure to the risk requires an equivalent change in the amount of capital needed to guard against this risk**

$$\rho(cX) = c\rho(X)$$

Coherent Property:
Translation Invariance

- **If we add some certain fixed amount to a random variable, the risk does not change**
- **This is the property of *translation invariance* and can be expressed as**

$$\rho(X+c) = \rho(X)+c$$

Risk Measure Example: Standard Deviation Principle

- **A simple and popular risk measure is the defined as the mean plus a fixed number of standard deviations, i.e.,** $\mu + k\sigma$ **, which is called the** ***standard deviation principle***
- **This risk measure is subadditive**

$$\mu_{X+Y} + k\sigma_{X+Y} = \mu_X + \mu_Y + k\sigma_{X+Y} \leq \mu_X + \mu_Y + k\sigma_X + k\sigma_Y$$
$$= \mu_X + k\sigma_X + \mu_Y + k\sigma_Y$$

- **This risk measure is positive homogeneous**

$$\mu_{c(X+Y)} + k\sigma_{c(X+Y)} = c\mu_{X+Y} + ck\sigma_{X+Y} = c(\mu_{X+Y} + k\sigma_{X+Y})$$

Risk Measure Example: Standard Deviation Principle

- Since standard deviation is not affected by a translation of the random variable, but the mean is shifted by exactly the translation, the standard deviation principle is translation invariant
- However, the standard deviation principle is not monotonic
- To see this, consider a bivariate random variable defined as

$$p(X,Y) = \begin{cases} 0.25 \text{ for } X = 0, Y = 4 \\ 0.75 \text{ for } X = 4, Y = 4 \end{cases}$$

- In this case

$$\mu_X = 3, \mu_Y = 4, \sigma_X = \sqrt{3}, \sigma_Y = 0$$

- Note that even though $X \leq Y$ we have that

$$\mu_X + \sigma_X = 3 + \sqrt{3} > 4 = 4 + 0 = \mu_Y + \sigma_Y$$

Percentile Funding is Not a Coherent Risk Measure!

- **In the special case of normally distributed random variables, VaR is coherent**
  - *However in general, VaR is not a coherent risk measure.*
- **VaR is monotonic, translation invariant, and positive homogeneous, but it is not subadditive!**
- **This is an important failure since the highly touted portfolio effect relies heavily on subadditivity, which does not apply to percentile funding!**
- **As an example, consider two independent Pareto distributed random variables, with**

$$\alpha = \frac{1}{2} \quad \text{and} \quad F(x) = 1 - x^{-1/2}$$

Non-Subadditivity Example

Then by convolution

$$Pr(X+Y \le z) = \int_1^{z-1} \int_1^{z-x} Pr(X=x)Pr(Y=y)\,dy\,dx$$

$$= \int_1^{z-1} \frac{1}{2} x^{-3/2} [-y]_1^{z-x} dx$$

$$= \int_1^{z-1} \frac{1}{2} x^{-3/2} \left(1 - (z-x)^{-1/2}\right) dx$$

$$= \int_1^{z-1} \frac{1}{2} x^{-3/2} dx \quad \frac{1}{2} \int_1^{z-1} x^{-3/2} (z \quad x)^{-1/2} dx$$

$$= \left[-x^{-1/2}\right]_1^{z-1} - \frac{1}{2} \int_1^{z-1} x^{-3/2} (z-x)^{-1/2} dx$$

Non-Subadditivity Example (2)

$$= 1 - \frac{1}{\sqrt{z-1}} - \frac{1}{2}\int_1^{z-1} x^{-3/2}(z-x)^{-1/2}\,dx$$

In order to integrate the remaining expression, set

$$u = \sqrt{x}$$

and thus

$$u = \frac{1}{2\sqrt{x}}dx$$

Therefore (ignoring integration limits for now),

$$1 - \frac{1}{\sqrt{z-1}} - \frac{1}{2}\int x^{-3/2}(z-x)^{-1/2}\,dx = 1 - \frac{1}{\sqrt{z-1}} - \int x^{-3/2}(z-x)^{-1/2}\,dx$$

Non-Subadditivity Example (3)

Now set

$$u = \sqrt{z}sin(s)$$

and thus

$$du = \sqrt{z}cos(s)ds$$

Then

$$\sqrt{z - u^2} = \sqrt{z - zsin^2(s)} = \sqrt{z}cos(s)$$

and

$$s = sin^{-1}\left(\frac{u}{\sqrt{z}}\right)$$

Non-Subadditivity Example (4)

Therefore

$$1-\frac{1}{\sqrt{z-1}}-\int\frac{1}{u^2\sqrt{z-u^2}}du=1-\frac{1}{\sqrt{z-1}}-\int\frac{csc^2(s)}{z}ds$$

$$=1-\frac{1}{\sqrt{z-1}}-\left[\frac{cot(s)}{z}\right]+c$$

where c is an arbitrary constant.

Note that

$$\cot(s)=\frac{\cos(s)}{\sin(s)}=\frac{\cos\left(\sin^{-1}\left(\frac{u}{\sqrt{z}}\right)\right)}{\frac{u}{\sqrt{z}}}=\frac{\sqrt{\cos^2\left(\sin^{-1}\left(\frac{u}{\sqrt{z}}\right)\right)}}{\frac{u}{\sqrt{z}}}$$

Non-Subadditivity Example (5)

$$= \frac{\sqrt{1 - sin^2\left(sin^{-1}\left(\frac{u}{\sqrt{z}}\right)\right)}}{\frac{u}{\sqrt{z}}} = \frac{\sqrt{1 - \frac{u^2}{z}}}{\frac{u}{\sqrt{z}}}$$

Thus

$$1 - \frac{1}{\sqrt{z-1}} - \left[\frac{cot(s)}{z}\right] = 1 - \frac{1}{\sqrt{z-1}} - \frac{\sqrt{1 - \frac{u^2}{z}}}{z\frac{u}{\sqrt{z}}} = 1 - \frac{1}{\sqrt{z-1}} - \frac{\sqrt{1 - \frac{u^2}{z}}}{u\sqrt{z}}$$

Since $u = \sqrt{x}$

$$1 - \frac{1}{\sqrt{z-1}} - \frac{1}{2}\int_1^{z-1} x^{-3/2}(z-x)^{-1/2}dx = 1 - \frac{1}{\sqrt{z-1}} + \left[\frac{\sqrt{1 - \frac{x}{z}}}{\sqrt{xz}}\right]_1^{z-1}$$

Non-Subadditivity Example (6)

$$= 1 - \frac{1}{\sqrt{z-1}} + \left[\frac{\sqrt{z-x}}{z\sqrt{x}}\right]_1^{z-1}$$

$$= 1 - \frac{1}{\sqrt{z-1}} + \frac{1}{z\sqrt{z-1}} - \frac{\sqrt{z-1}}{z}$$

$$= 1 - \frac{2\sqrt{z-1}}{z}$$

That is

$$Pr(X+Y \leq z) = 1 - \frac{2\sqrt{z-1}}{z}$$

Non-Subadditivity Example (7)

Note however that

$$Pr(2X \le z) = Pr\left(X \le \frac{z}{2}\right) = 1 - \left(\frac{z}{2}\right)^{-\frac{1}{2}} \ge 1 - \frac{2\sqrt{z-1}}{z} = Pr(X + Y \le z)$$

when $z \ge 2$

From this it follows that

$$VaR_\alpha(X+Y) > VaR_\alpha(2X) = VaR_\alpha(X) + VaR_\alpha(Y)$$

A Second Non-Subadditivity Example

- **Two projects in a program with 50% chance of a funding shortfall for the program**
- **If the shortfall occurs, it will impact the project with the lesser amount of progress, and assume it is equally likely that each project's funding will be cut**
- **Thus there is a 25% chance of a shortfall occurring to either project. A shortfall in funding will mean that funds will not be available when needed, leading to delays, which will in turn lead to cost growth**
  - **The impact of the shortfall will be to increase the cost by $20 million.**
- **Budget for each project is $100 million and the funding shortfall is the only risk**
- **70th percentile for each project is $100 ($120 million is the 75th percentile). So $VaR_{0.70}$ funding for the projects is $200 million**
  - **But $VaR_{0.70}$ funding for the program is $220 million, since the 50th percentile for the program is $220 million**

The Impact of Non-Subadditivity

- **When projects are not subadditive, the overall portfolio is riskier than the sum of the individual projects**
- **Smart (Refs. 4 and 5) showed conclusively that for subadditive projects the portfolio effect is at best minimal. And now we have that percentile funding it is possible to have a reverse portfolio effect!**
- **This is clearly an undesirable property for a risk measure and calls into question the use of this measure in policy**

Conditional Tail Expectation

- **We have shown that percentile funding is not effective risk management**
- **A better measure that overcomes the drawbacks of percentile funding is *Conditional Tail Expectation (CTE)***
  - **This is defined as the amount of cost growth to expect given that cost has exceeded a specified amount**

$$E\left[X \mid X > Q_\alpha\right]$$

where Q_a is a specified percentile

- **For example, $Q_{0.95}$ is the 95th percentile. This risk measure is also called the "Tail Value at Risk" and the expected shortfall (Ref. 10)**

Tail Value at Risk

- **Conditional Tail Expectation (CTE) is also referred to as Tail Value at Risk, since**

$$CTE_\alpha = E\left[X \mid X > Q_\alpha\right] = \frac{1}{1-F(Q_\alpha)}\int_{Q_\alpha}^{1} xf(x)dx = \frac{1}{1-\alpha}\int_{\alpha}^{1} VaR_u(X)du$$

- **Note that for a Normal distribution**

$$VaR_\alpha(X) = \mu + \sigma\Phi^{-1}(\alpha)$$

Conditional Tail Expectation
Normal Distribution

Note that for a Normal distribution

$$CTE_\alpha(X) = \mu + \sigma E\left[\frac{X-\mu}{\sigma}\middle|\frac{X-\mu}{\sigma} \geq Q_\alpha\left(\frac{X-\mu}{\sigma}\right)\right]$$

$$\begin{aligned} E\left[\frac{X-\mu}{\sigma}\middle|\frac{X-\mu}{\sigma} \geq Q_\alpha\left(\frac{X-\mu}{\sigma}\right)\right] &= \frac{1}{1-\alpha}\int_{\Phi^{-1}(\alpha)}^{\infty} x\phi(x)\,dx \\ &= \frac{1}{1-\alpha}\left[-\phi(x)\right]_{\Phi^{-1}(\alpha)}^{\infty} \\ &= \frac{\phi\left(\Phi^{-1}(\alpha)\right)}{1-\alpha} \end{aligned}$$

where ϕ represent the standard normal density function and Φ^{-1} represents the inverse of the cumulative normal distribution

Conditional Tail Expectation
Lognormal Distribution

For a lognormal distribution

$$CTE_\alpha = VaR_\alpha + \frac{E[X] - E[X \wedge VaR_\alpha]}{1-\alpha}$$

Note that

$$E[X \wedge VaR_\alpha] = \int_0^{VaR_\alpha} \frac{1}{\sqrt{2\pi}\sigma} exp\left(-\frac{1}{2}\left(\frac{lny-\mu}{\sigma}\right)^2\right) dy$$

and setting

$$z = \frac{lny - \mu - \sigma^2}{\sigma}$$

the integral simplifies to

$$exp\left(\mu + \frac{\sigma^2}{2}\right) \int_{-\infty}^{\frac{lnVaR_\alpha - \mu - \sigma^2}{\sigma}} \frac{1}{\sqrt{2\pi}} exp\left(-\frac{1}{2}z^2\right) dz = E[X]\left[\Phi\left(\frac{\ln VaR_\alpha - \mu - \sigma^2}{\sigma}\right)\right]$$

Conditional Tail Expectation
Lognormal Distribution (2)

Thus the CTE for the lognormal distribution can be written as

$$\mathbf{VaR}_{\alpha} + \frac{\mathbf{E[X]} - \mathbf{E[X]}\left[\Phi\left(\frac{\ln \mathbf{VaR}_{\alpha} - \mu - \sigma^2}{\sigma}\right)\right] - \mathbf{VaR}_{\alpha}(1-\alpha)}{1-\alpha}$$

$$= \frac{\mathbf{E[X]}\left[1 - \Phi\left(\frac{\ln \mathbf{VaR}_{\alpha} - \mu - \sigma^2}{\sigma}\right)\right]}{1-\alpha}$$

where Φ is the cumulative normal distribution function

Conditional Tail Expectation
Example

For example, for a single project for which cost risk has been modeled as a lognormal distribution with mean equal to $100 million and standard deviation equal to $50 million, $\mu = 4.49$, $\sigma = 0.72$, and the 70th percentile is equal to

$$e^{4.49+z_{0.70}\cdot 0.72} \approx \$114.6\,\textit{million}$$

Thus

$$CTE_{0.70} = 100 \cdot \frac{1-\Phi\left(\dfrac{\ln 114.6 - 4.49 - 0.47^2}{0.47}\right)}{1-0.7} \approx \$159.7\ million.$$

CTE is 40% more than the 70th percentile

Conditional Tail Expectation is a Coherent Risk Measure

- **Most of the coherence properties follow naturally from properties of percentiles, in particular positive homogeneity and translation invariance**
  - **Monotonicity naturally follows, since if X is always less than or equal to Y, the conditional expected value of X greater than some fixed value will always be less than the conditional expected value of Y for that same fixed value**
- **For subadditivity, note that**

$$CTE_\alpha(X) + CTE_\alpha(Y) - CTE_\alpha(X+Y)$$

$$= E[X|X > Q_\alpha] + E[Y|Y > Q_\alpha] - E[X+Y|X+Y > Q_\alpha]$$

$$= E[X|X > Q_\alpha] - E[X|X+Y > Q_\alpha] + E[Y|Y > Q_\alpha] - E[Y|X+Y > Q_\alpha]$$

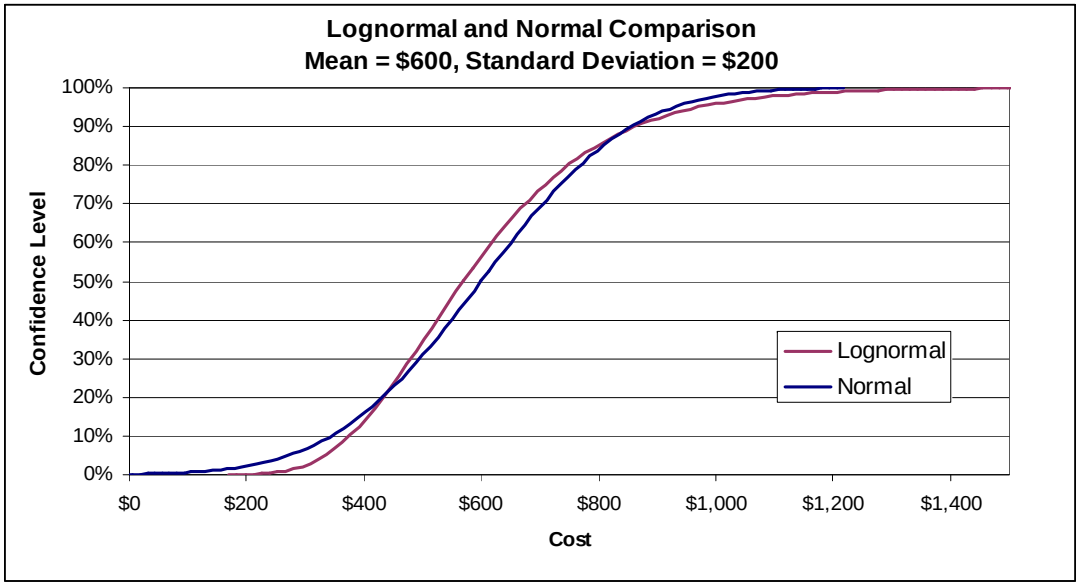
which can easily seen by definition to be greater than zero since both expected value differences are nonnegative, proving subadditivity

Solving the Lognormal Paradox with Conditional Tail Expectation

- **Another problem with VaR, or percentile funding that is solved by CTE is what the author has termed the "Lognormal Paradox"**
- **As discussed in Ref. 4, with funding levels at or below the 84th percentile, for a common mean and standard deviation, a normal distribution will require more funding than a lognormal distribution**
  - **Even though the lognormal has a heavier right tail, and hence is riskier. Contrary to common sense, which tell us that riskier events should require greater funding.**
- **Not an issue for CTE because it takes into account the right tail of the cost risk distribution**

Example of the Lognormal Paradox for Percentile Funding

- As is evident from the figure, for percentiles between the 23rd and 84th percentiles, normal distribution has higher percentile levels than the lognormal distribution
  - Despite the fact that the means and standard deviations are the same and the lognormal is riskier than the normal distribution, all else being equal



CTE and Percentile Funding Comparison

- **Note that CTE does not suffer from the Lognormal paradox**

	Mean = $600, Standard Deviation = $200								
	α =	50.0%	60.0%	70.0%	80.0%	90.0%	95.0%	99.0%	99.9%
VaR	Normal	$600	$651	$705	$768	$856	$929	$1,065	$1,218
	Lognormal	$569	$618	$675	$748	$863	$971	$1,211	$1,552
CTE	Normal	$760	$793	$832	$880	$951	$1,013	$1,133	$1,273
	Lognormal	$753	$793	$842	$908	$1,016	$1,120	$1,359	$1,704

Calculating CTE

- **CTE is easy to calculate – we have given formulas that can easily be implemented in a spreadsheet when total risk is represented by a normal or lognormal distribution and when Monte Carlo simulation is used for cost risk estimation, it is easy to estimate CTE from the trial data**

CTE for Simulation Data

- **For example in a 10-trial Monte Carlo simulation of a normal distribution with mean equal to $600 and standard deviation equal to $200, whose trial values are shown in the table, the 70th percentile represents values above $687.21**
- **To calculate $CTE_{0.70}$ we take the mean of the three values $732.19, $755.82, and $779.58, which is equal to $755.86**

1	379.69
2	450.73
3	451.91
4	504.46
5	548.09
6	661.94
7	687.21
8	732.19
9	755.82
10	779.58

Conditional Tail Expectation Is Widely Used

- **Conditional tail expectation was introduced in the late 1990s and quickly became the preferred standard for setting liabilities for insurance settings**
  - **In Canada, the "actuarial Standards of Practice promulgate the use of the CTE whenever stochastic methods are used to set balance sheet liabilities" (Ref. 12)**
  - **It is also the basis for the Swiss Solvency Test (Ref. 13), which forms a major part of Swiss insurance policy**
  - **And the National Association of Insurance Commissioners recommends setting reserves using CTE (Ref. 14)**

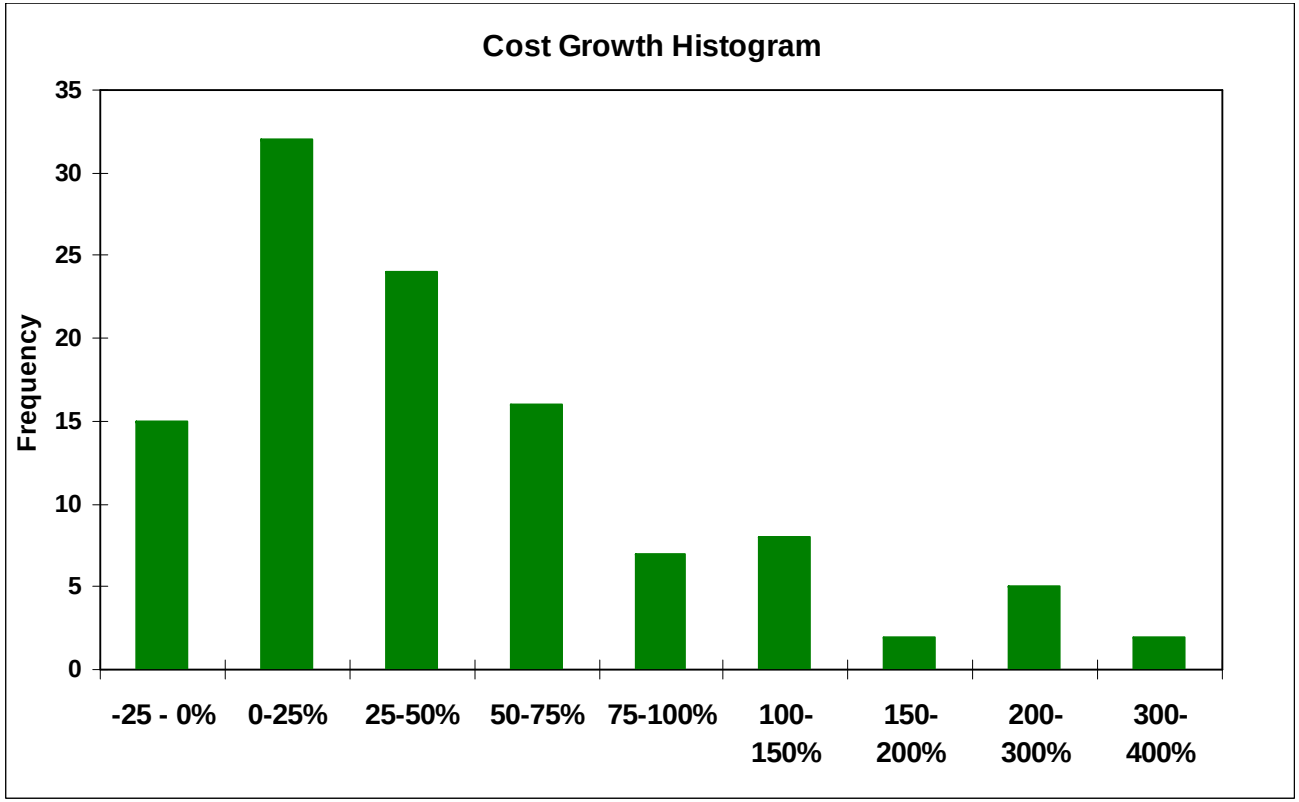
Practical Impact of Conditional Tail Expectation

- **Percentile funding is still being implemented as a practical policy by government agencies**
- **The question still outstanding is whether or not this policy will be effective in containing cost growth - fewer missions overall should experience cost growth, but what about those who do?**
- **As we have shown, percentile funding is not a true risk management policy as additional funding, perhaps a significant amount, will be required much of the time, as much as 30% or more**
- **But how much additional cost growth should be expected when these overruns occur?**

Practical Impact of Conditional Tail Expectation (2)

- To gain an understanding of how much additional funding will be required for percentile funding in practice, historical cost growth data are used
- As discussed by Smart (Ref. 5), for a data set of cost growth for 112 recent NASA missions, the minimum cost growth was -25.2% for Super Light Weight Tank (SLWT), an upgrade for the Shuttle Program from a more traditional aluminum structure to aluminum-lithium
  - The maximum cost growth among the missions studied was 385%
- The average cost growth for all missions was 53.0%, with median growth equal to 32.1%
  - The difference between the mean and median indicates a high degree of positive skew in the data
  - Seventeen missions had cost growth in excess of 100%

Cost Growth Histogram



Cost Growth and Cost Risk

- **Cost risk is the probability that an estimate will exceed a specified amount, such as $100 million or $150 million**
  - **Cost growth and cost risk are thus intrinsically related**
- **Historical cost growth provides an excellent means for determining the overall level of risk for cost estimates**
  - **For example, if 95% of past programs have experienced less than 100% growth, we should expect that the ratio of actual cost to the initial estimate should be less than 100% with 95% confidence**

Cost growth is the impact of cost risk in action

Conditional Tail Expectation for 70th Percentile Funding

- **Note that for the case of NASA, with 70th percentile funding and a coefficient of variation implied by the data of 100%, for missions that experience cost overruns beyond the 70th percentile, on average an additional 89% funding will be needed to complete such projects**

Coefficient of Variation	Budget Set at 30th	40th	50th	60th	70th	80th	90th
20%	23.6%	20.5%	18.0%	15.9%	14.0%	12.2%	10.2%
30%	38.0%	32.7%	28.5%	25.0%	21.9%	19.0%	15.8%
40%	54.1%	46.1%	40.0%	34.9%	30.4%	26.2%	21.6%
50%	72.0%	60.9%	52.4%	45.5%	39.4%	33.7%	27.7%
60%	91.6%	76.8%	65.7%	56.7%	48.8%	41.5%	33.9%
70%	112.7%	93.8%	79.7%	68.3%	58.5%	49.5%	40.1%
80%	136.0%	112.0%	94.4%	80.5%	68.6%	57.6%	46.4%
90%	160.0%	131.0%	110.0%	93.0%	78.8%	65.9%	52.7%
100%	185.0%	151.0%	125.5%	105.8%	89.2%	74.2%	59.0%

Conditional Tail Expectation for 70th Percentile Funding

- **Here there be dragons indeed - this is a sobering amount**
  - **30% of the time, approximately 90% more money will be needed, and indicates that even if risk models are calibrated to empirical cost growth experience, the average project should expect to experience 27% growth**
  - **And 30% of the time, missions will continue to experience an embarrassing amount of cost growth**
  - **While an improvement over the current 53% average growth percentile funding will not be the hoped-for panacea but only a band aid where major surgery is required**

Summary

- **Percentile Funding is problematic and an ineffective as a risk management policy**
  - **Provides only an indicator of a problem**
  - **Does not take into account tail behavior**
    - **Events with widely different risk profiles may receive same funding**
  - **Not coherent, no portfolio effect guaranteed and the reverse may occur**
  - **Riskier events may receive less funding (paradox)**
  - **Will not lead to a solution for cost growth**

Summary (2)

- **Conditional tail expectation is a better risk measure to develop a sound and effective risk management policy**
  - **Takes into account tail behavior**
    - **Avoids paradoxical behavior of percentile funding**
  - **Coherent (no reverse portfolio effect)**
  - **It is more consistent and logical than percentile funding**
  - **While more sophisticated it is easy to calculate and can be communicated to management**

Dilbert and Percentile Funding

References

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